

The Perfect Quark-Gluon Vertex Function *

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We evaluate a perfect quark-gluon vertex function for QCD in coordinate space and truncate it to a short range. We present preliminary results for the charmonium spectrum using this quasi-perfect action.

Approximately perfect lattice actions have a potential to suppress lattice artifacts much more than the $O(a)$ improvement, which is very fashionable at this conference. However, it is still unclear how well this more sophisticated improvement program can be applied to QCD. Here we discuss our recent progress in the construction of a quasi-perfect action for QCD, and show preliminary results of its application to heavy quarks.

For free fermions we have derived extremely local perfect lattice actions [1] of the form,

$$S[\bar{\Psi}, \Psi] = \sum_{x,y} \bar{\Psi}_x [\gamma_\mu \rho_\mu(y-x) + \lambda(y-x)] \Psi_y.$$

The couplings decay exponentially (in $d > 1$). We truncate them to a unit hypercube by means of 3-periodic boundary conditions. The resulting “hypercube fermion” has still excellent spectral and thermodynamic properties [2]. As an example, we give the couplings for $m = 1$ in Table 1 and show the dispersion relation in Fig. 1. It is very close to the continuum dispersion and approximates rotational symmetry very well.

Expansion to $O(gA_\mu)$ for full QCD yields

$$\begin{aligned} S[\bar{\Psi}, \Psi, A_\mu] &= S[\bar{\Psi}, \Psi] + S[A_\mu] + gV[\bar{\Psi}, \Psi, A_\mu] \\ V[\bar{\Psi}, \Psi, A_\mu] &= \frac{1}{(2\pi)^{2d}} \int_{B^2} dp dq \bar{\Psi}^i(-p) V_\mu(p, q) \\ &\quad A_\mu^a(p-q) \lambda_{ij}^a \Psi^j(q). \end{aligned}$$

*Based on a poster presented by K. Orginos at LAT97.

$y-x$	$\rho_1(y-x)$	$\lambda(y-x)$
(0000)	0	1.26885069540
(1000)	0.05457967484	-0.03008271460
(1100)	-0.01101007028	-0.01082956270
(1110)	-0.00325481234	-0.00471575763
(1111)	-0.00120632489	-0.00221240767

Table 1

The “hypercube fermion” couplings at $m = 1$.

The perfect action for free gluons, $S[A_\mu]$, has also been discussed in [1]. Moreover, we found an explicit but complicated expression for the *perfect quark-gluon vertex function* $V_\mu(p, q)$.

Numerically the vertex function can be evaluated and transformed to c-space, where it is represented by a set of link couplings, which depend on the fermion positions.

We truncate the $O(gA_\mu)$ perfect action and parameterize it in a gauge invariant form:

1) Take the mean value of the link couplings over short lattice paths connecting $\bar{\Psi}$ and Ψ .

2) Re-scale the path and plaquette couplings such that their sum amounts to $\lambda(r)$ for the scalar terms ($\propto \mathbb{1}$), $\rho_\mu(r)$ for the vector terms ($\propto \gamma_\mu$), $s(m) = (m/\hat{m})^2 [1/m - 1/\hat{m}]$, $\hat{m} \doteq e^m - 1$ for the plaquette couplings ($\propto \sigma_{\mu\nu}$) [2] and a value $s_1(m)$ – obtained from a low \vec{p} expansion – for the terms $\propto \gamma_\mu \gamma_\nu \gamma_\rho$.

For our first experiments we impose a tough truncation and arrive at the following parameterization of $V_\mu(x, y)$:

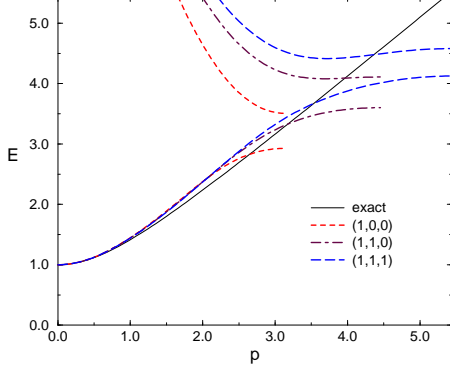


Figure 1. The dispersion relation of the hypercube fermion at $m = 1$ in various directions.

		$m = 0$	$m = 1$	$m = 2$
\mathbb{I}	L_1	-0.01315	-0.00528	-0.00184
	L_3^s	-0.00793	-0.00413	-0.00169
	L_2	-0.01502	-0.00541	-0.00163
	L_3	-0.00266	-0.00079	-0.00019
	L_4	-0.00035	-0.00009	-0.00002
γ_μ	L_1	0.02982	0.01086	0.00338
	L_3^s	0.01784	0.00729	0.00253
	L_2	0.01604	0.00551	0.00158
	L_3	0.00184	0.00054	0.00013
	L_4	0.00020	0.00005	0.00001
$\sigma_{\mu\nu}$	c_0	0.07283	0.02207	0.00556
	c_1	0.05217	0.01333	0.00285
$\gamma_\mu\gamma_\nu\gamma_\rho$	s_1	0.05457	0.01335	0.00268

For the scalar term, L_1 is a path consisting of one link, L_3^s is a staple and L_2 , L_3 , L_4 are the shortest paths connecting $\bar{\Psi}$, Ψ separated by a 2d, 3d, 4d diagonal in the unit hypercube, respectively. The same holds for the vector term, if the fermions are separated in the μ direction. The standard clover-like plaquette coupling is called c_0 , and c_1 refers to the case where $\bar{\Psi}$, Ψ are separated by one lattice spacing; it is the coupling to the plaquettes attached to their connecting link. Finally, if the fermions are separated by $\hat{\rho}$, then we couple them to the plaquettes in the perpendicular μ, ν planes (touching the fermions), and to the connecting link, by s_1 .

A low \vec{p} expansion leads to a Hamiltonian of

the form

$$H = m_s + \frac{1}{2m_{kin}} \vec{p}^2 + \frac{1}{2m_B} \vec{\Sigma} \vec{B} + \dots$$

where \vec{B} is assumed const. in time, and $\Sigma_k = \epsilon_{ijk} \sigma_{ij}/2$, [3]. Here m_s is the static, m_{kin} the kinetic and m_B the magnetic mass. In the continuum they all coincide, and for the hypercube fermion $m_s = m$.

We saw before that $m_{kin}(m_s)$ is drastically improved for the hypercube fermion, compared to the Wilson fermion [2]. Of particular interest for the hyperfine splitting is $m_B(m_s)$, see Fig. 2.

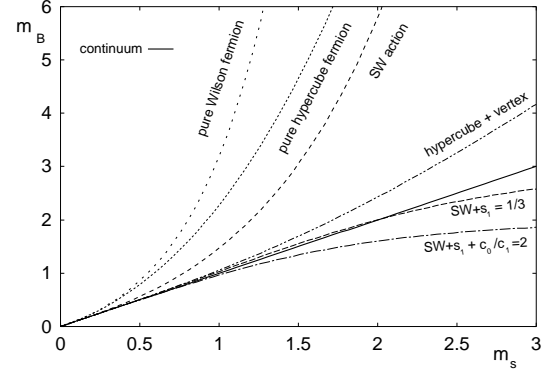


Figure 2. The magnetic vs. static mass.

Remarkably, the Wilson fermion can be corrected up to the quartic order by the parameters we introduced before:

pure Wilson fermion: $m_B = m_s + m_s^2 + \dots$

Sheikholeslami-Wohlert action ($c_0 = 1$): $m_B = m_s + \frac{2}{3}m_s^3 + \dots$ (and $m_B = m_{kin}$).

" " + $s_1 = \frac{1}{3}$: $m_B = m_s + \frac{1}{6}m_s^4 + \dots$

split clover term into $c_0 = \frac{2}{3}$, $c_1 = \frac{1}{3}$: $m_B = m_s - \frac{4}{45}m_s^5 + \dots$

We now present preliminary results on *charmionium spectroscopy* using the fermionic action presented above and the Wilson gauge action. We simulate quenched at $\beta = 5.6$, the lattice spacing is 0.24 fm and the size $8^3 \times 16$. We include 30 lattices and consider the meson dispersion relation as well as the spectrum of the low lying states. The latter was evaluated at $m = 0$ and $m = 1$. We then interpolate to the mass of η_c (2.98 GeV), which corresponds to $m \simeq 0.9$.

Regarding the dispersion relation we compute the "speed of light" from $cp = \sqrt{E^2 - M^2}$, see

Fig. 3, and obtain $c = 1.04 \pm 0.05$. The mesonic dispersion is impressively accurate even for large momenta.

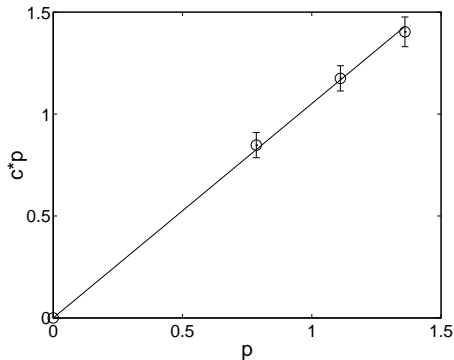


Figure 3. The mesonic dispersion relation.

In Fig. 4 we show the charmonium spectrum, including the 1s and 2s states of η_c and J/ψ . The 2s-1s splitting looks fine, but the hyperfine splitting is clearly too small (10.3 MeV). We also note that the entirely new term $\propto \gamma_\mu \gamma_\nu \gamma_\rho$ has a positive influence on the spectrum. Without of it, both types of splitting decrease by about one fourth.

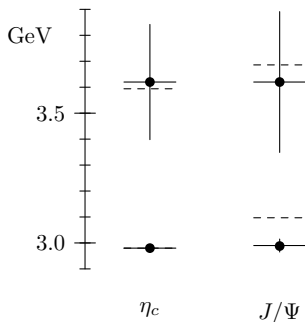


Figure 4. The charmonium spectrum. The experimental values are dashed, and only the ground state of η_c is fitted.

Finally we consider one more point, $\beta = 5.5$, where η_c corresponds to $m \simeq 1.1$. Now the lattice is even coarser, $a = 0.28$ fm. The hyperfine splitting decreases to 7.4 MeV, such that its slope for decreasing lattice spacing points up, as it should. It is hardly justified to extrapolate the connection of these two points by a straight line down to the continuum limit. If one does so nevertheless one obtains $M_{J/\psi} - M_{\eta_c} \sim 28$ MeV. The

reference value here is perhaps not so much the experimental value of 118 ± 2 MeV, but rather the relativistic quenched results ~ 70 MeV; for a review see [4]. NRQCD reported a larger value for some time, but more recent observations indicate that the NR expansion is not really applicable to charmonium [5].

To *summarize* our first experience with the hypercube fermion plus truncated vertex function, we observed that properties related to the restoration of Lorentz symmetry are strongly improved: fermionic dispersion, kinetic mass and mesonic dispersion. However, properties related to the magnetic interaction are only successful in part: m_B is significantly improved, but the hyperfine splitting is too small. The 2s-1s splitting looks fine. On the other hand, the mass is still strongly renormalized.

Since renormalization due to $O(A^2)$ is not incorporated yet, we expect tadpole improvement to amplify the hyperfine splitting and to reduce the dependence on the lattice spacing. We also hope to further improve the parameterization.

But even if this action should not be satisfactory in a direct application, we expect it to be a good starting point for the non-perturbative multigrid improvement program described in [2]. Furthermore it helped to identify the significant non-standard terms, in particular the c_1 and the s_1 term. Their dominating rôle in the improvement is a reliable observation, even if the exact coefficients may still change.

We thank R. Edwards for allowing us to use the SZIN package and his fitting routines. Much of the computation was done at the Theoretical Physics Computing Facility at Brown University.

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